MISCELLANEOUS.

104. Proposed by HARRY S. VANDIVER, Bala, Pa.

A Theorem of Fermat. The area of a right angled triangle with commensurable sides cannot be a square number. [Cf. Chrystal's *Algebra*, Vol. II., page 535.]

Solution by L. C. WALKER, A. M., Professor of Mathematics, Petaluma High School, Petaluma, Cal.

Since the sides of the right-angled triangle may have a common factor, we let af, bf, and cf represent its sides, where a, b, c are prime to each other. Then we have $a^2+b^2=c^2$......(1).

For the area, $\frac{1}{2}abf^2$, of the triangle to be a square, $\frac{1}{2}ab$ must be a square.

In (1) either a or b must be even and the other odd, because the sum of two odd squares can not be a square. Assume a even and b odd. Then for $\frac{1}{2}ab$ to be a square, b must be a square, and a must be double a square.

Let $a=2m^2$, $b=n^2$, and substitute these values in (1); then we have

$$(2m^2)^2+(n^2)^2=c^2......(2).$$

Now set $2m^2=2pq$, and $n^2=p^2-q^2$; or

$$n^2+q^2=p^2......(3).$$

In (3) p is odd, and we will assume q even. Let $q=2\alpha\beta$, $p=\alpha^2+\beta^2$, and we find $m=2\alpha\beta(\alpha^2+\beta^2)$......(4).

For $2\alpha\beta$ to be a square, we will assume $\alpha=2m_1^2$ and $\beta=n_1^2$. Substituting these values in (4), we obtain

$$m^2=4m_1^2n_1^2(4m_1^4+n_1^4)......(5).$$

In order that the right member of (5) may be a square, we must have

$$4m_1^4+n_1^4=c_1^2,$$

say; which is of the same form of (2). But c_1^2 is less than c^2. Proceeding in exactly the same way, we can reduce c^2 indefinitely. By our hypothesis c^2 can not be reduced to zero nor less. Hence, results *Fermat's Theorem.*

PROBLEMS FOR SOLUTION.

ARITHMETIC.

154. Proposed by J. SCHEFFER, A. M., Hagerstown, Md.

Suppose there is a meadow of 8 acres in which the grass grows uniformly, and that 21 oxen could eat up the whole pasture in 6 weeks, or 18 oxen in 9 weeks; what number of oxen diminished by the removal of 9, at the end of 14 weeks, could eat it up in 18 weeks?